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FINAL REPORT

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3. TITLE OF PROJECT:	Robust Optimal PWM and PAM Controllers for Uncertain Aerospace Systems
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6. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT DURING THIS REPORTING PERIOD:

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7. SUMMARY OF RESEARCH RESULTS:

Most physical processes and complex systems are formulated by hybrid uncertain systems that consist of mixed continuous and discrete uncertain subsystems. The uncertainties in these systems arise from unmodeled dynamics, parameter variations, sensor noises, actuator constraints, etc. These variations do not follow any of the known probability distributions in general, and are most often quantified in terms of amplitude and/or frequency bounds. Hence, practical systems are most suitably represented by hybrid interval models with bounded parameters, disturbances and noise inputs. At present, no effective method and software are available for digital modeling and digital simulation of such hybrid interval systems. With the aid of interval arithmetic, we have developed new interval methods for finding an equivalent discrete-time or continuous-time interval model for a hybrid interval system. As a result, the well-developed theories and methods in either the discrete-time domain or the continuous-time domain can be effectively applied to the analysis and design of hybrid uncertain systems.

Moreover, to improve significantly the qualitative and quantitative properties of an uncertain system represented by a continuous-time uncertain framework, analogue robust control design methods, such as the continuous-time H_2/H_∞ robust control methods, have been developed in the past. With the rapid advances in digital technology and computers, the resulting analogue robust controller is often required to be implemented using a digital controller for better reliability, lower cost, more flexibility and better performance. The process of converting an analogue controller to an equivalent digital controller, so that the performance of the digitally controlled hybrid uncertain system closely matches that of the original continuous-time controlled uncertain system for a relatively longer sampling period, is called "digital redesign". At present, no effective method is available for digital redesign of such hybrid uncertain systems. We have recently developed a new digital redesign method for robust digital control of continuous-time interval systems. The new digital controllers can be classified into two types: The PAM (Pulse-Amplitude Modulated) controller and the PWM (Pulse-Width Modulated) controller. The PAM controller, which produces a series of piecewise-constant continuous pulse having a variable amplitude and a fixed or variable width, is commonly used in digital control systems of all types, while the PWM controller, which provides a series of discontinuous pulse with a fixed amplitude and a variable width, has become popular in military and industry for on-off control of DC power converters, stepper motors widely used in robotics, satellite station-keeping (with on-off rejection jets), etc. Utilizing the practically implementable digital controllers, the resulting dynamic states of the digitally controlled sampled-data interval systems are able to closely match those of the original analogously controlled continuous-time interval systems.

Also, the PAM and PWM controllers newly developed by us have been successfully implemented and tested on the rigid and flexible body dynamics of the Hubble Space Telescope extended on the Shuttle's Remote Manipulator by using the high-fidelity non-linear Interactive On-orbit Simulation/Draper RMS Simulation (IOS/DRS). The details of the development and simulation results have been presented at the AIAA Guidance, Navigation and Control Conference held in Baltimore, August 1995, and the SPIE's Symposium on Mathematics and Control in Smart Structures held in San Diego, Febuary 1996.

Based on the research results in the period of June 15, 1994 to June 15, 1996, twenty technical papers have been published in the referred journals and listed as follows:

(1). Tsai, J.S., C.T. Wang, and L.S. Shieh, "Model Conversion and Digital Redesign of Singular Systems," *Journal of the Franklin Institute*, Vol. 330, No. 6, pp. 1063-1086, November 1993.

Abstract

Design procedures are proposed for model conversion and digital redesign of a singular system, which is controllable at finite and impulsive modes. In order to attain a standard regular problem, we use some techniques to decompose the singular system into a reduced-order regular subsystem and a nondynamic subsystem. As a result, some well-known design methodologies for a regular system can be applied to the reduced-order regular subsystem. Finally, we transform the results obtained back to those of the original coordinate system.

(2). Wei, C.P., J.S. Tsai, and L.S. Shieh, "Design of Optimal Digital Servo Controllers for Continuous-time Input-delay Systems," *Journal of Control Systems and Technology*, Vol. 1, No. 4, pp. 267-283 1993.

Abstract

This paper deals with the design method of an optimal robust digital servo controller with two degrees of freedom for a continuous-time plant with an input delay. The proposed digital controller includes not only state-feedback terms from the plant and the servocompensator but also state-feedforward terms from the reference and disturbance signals. We introduce a discrete-time performance index to optimize the feedback and feedforward gains and use state reconstructors to realize the servo controller. As a result, the sampling output of the digital system tracks the sampling reference signal in the presence of the disturbance. In addition, the use of the optimal feedforward compensation leads to a satisfactory transient response to the reference signal.

(3). Koc, C.K., B. Bakkaloglu, and L.S. Shieh, "Computation of the Matrix Sign Function Using Continued Fraction Expansion," *IEEE Trans. Automatic Control*, Vol. 39, No. 8, pp. 1644-1647, August, 1994.

Abstract

We describe an algorithm which computes the sign function of a complex matrix by using the continued fraction expansion of the inverse of the principal square root function at each step of the iteration. We show that the algorithm iteratively computes globally convergent main diagonal pade approximants. The proposed algorithm avoids computing large matrix powers and performs fewer matrix inversions than Newton's method. The algorithm is multiplication-rich and particularly suitable for implementation on vector and parallel computers. The stability analysis of the algorithm suggests that the errors introduced during a step are either suppressed or have limited effect on the next step. Finally, we summarize the results of our experiments on computing the sign function of certain matrices.

(4). Yuan, Z., Z. Chen, and L.S. Shieh, "Robustness of Reduced-order Multivariable State-space Self-tuning Controller," *Control Theory and Applications*, Vol. 11, No. 3, pp. 277-286, June, 1994.

Abstract

In this paper, we present a quantitative analysis of the robustness of a reduced-order pole-assignment state-space self-tuning controller for a multivariable adaptive control system whose

order of the real process is higher than that of the model used in the controller design. The result of stability analysis shows that, under a specific bounded modelling error, the adaptively controlled closed-loop real system via the reduced-order state-space self-tuner is BIBO stable in the presence of unmodelled dynamics.

(5). Guo, T.Y., C. Hwang, and L.S. Shieh, "Model Reduction of Nonsquare Linear MIMO Systems Using Multipoint Matrix Continued-fraction Expansions," *Journal of the Franklin Institute*, Vol. 331B, No. 2, pp. 189-216, 1994.

Abstract

This paper deals with the multipoint Cauer matrix continued-fraction expansion (MCFE) for model reduction of linear multi-input multi-output (MIMO) systems with various numbers of inputs and outputs. A salient feature of the proposed MCFE approach to model reduction of MIMO systems with square transfer matrices is its equivalence to the matrix pade approximation approach. The Cauer second form of the ordinary MCFE for a square transfer function matrix is generalized in this paper to a multipoint and nonsquare-matrix version. An interesting connection of the multipoint Cauer MCFE method to the multipoint matrix pade approximation method is established. Also, algorithms for obtaining the reduced-degree matrix-fraction descriptions and reduced-dimensional state-space models from a transfer function matrix via the multipoint Cauer MCFE algorithm are presented. Practical advantages of using the multipoint Cauer MCFE are discussed and a numerical example is provided to illustrate the algorithm.

(6). Tsai, J.S.H., W.S. Chen, L.S. Shieh, "Realization, Design and Implementation of Row-Pseudoproper Left Matrix Fraction Descriptions with Impulsive Modes," *Computers Math. Applic.*, Vol. 28, No. 7, pp. 13-41, 1994.

Abstract

A novel and minimal realization algorithm is proposed for determining generalized state-space representation from a so-called row-pseudoproper left matrix fraction description (LMFD). The realized system with the state-space representation form is proved to be controllable and observable in the sense of [1,2] if the given row-pseudoproper MFD is left coprime. Besides, the proposed state feedback control law not only satisfies the optimal regional-pole-placement design for the realized generalized dynamical system, but also eliminates the impulsive terms in the state response of the closed-loop system. For practical consideration, an equivalent input-output feedback structure of the designed state-feedback controller is adopted. Based on the cascaded and/or parallel active RC networks with better sensitivity and stability properties, the resulting structure of the equivalent input-output feedback controller can be readily implemented based on proper subsystems.

(7). Tsai, J.S.H., C.P. Wei and L.S. Shieh, "Design of a Stable Digital Output Feedback Controller for Continuous-time Input-delay Plants," *International Journal of Systems Science* Vol. 25, No. 12, pp. 2187-2206, 1994.

Abstract

This paper presents a stable multirate output-feedback controller for digital control of a continuos-time input-delay system. It is shown that an arbitrary digital state feedback controller for an observable input-time-delay plant can be realized by using multirate sampled output data. The developed digital mulirate-output feedback controller enables us to achieve results similar to those of the state feedback controller, without constructing an observer.

(8). Tsai, J.S.H., F.C. Liu and L.S. Shieh, "Model Conversions of Uncertain Linear Time-delay Systems Using the Block-pulse Function Approach," *Journal of Control Systems and Technology*, Vol. 2, No. 2, pp. 127-136, 1994.

Abstract

This paper proposes a block-pulse function approach along with the digital redesign concept to obtain the model conversions of a continuous-time (discrete-time) uncertain linear time-delay system to an equivalent discrete-time (continuous-time) uncertain linear time-delay model. The proposed method allows the use of well-developed theorems and algorithms in the discrete-time (continuous-time) domain to indirectly solve continuous-time (discrete-time) domain problems. The continuous-time uncertain time-delay system is considered to be composed of continuous-time nominal matrices with uncertainties and an input part with pure delay time. The digital redesign concept is used to determine the discrete-time uncertainties from the continuous-time uncertain time-delay system, and vice versa. Moreover, the paper also proposes a method to estimate tighter bounds of discrete-time (continuous-time) structured uncertainties based on the given continuous-time (discrete-time) structured uncertainties. An example is given to demonstrate the effectiveness of the proposed method.

(9). Wang, S.G. and L.S. Shieh, "Robustness of Linear Quadratic Regulators with Regional-Pole Constraints for Uncertain Linear Systems," *Control-Theory and Advanced Technology*, Vol. 10, No. 4, Part 1, pp. 737-769, 1994.

Abstract

This Paper presents robust pole clustering bounds for uncertain linear systems with nominal linear quadratic regulators (LQRs). The pole clustering regions of interest are the vertical strip, the horizontal strip and the ring (including the circle as a special case) with a center at the real axis in the s-plane. The Raleigh principle, along with the conventional norm theory, instead of the commonly used Lyapunov-type approach, is utilized for determining the robustness bounds of the poles of the perturbed closed-loop systems. Also, a constraint is established to verify whether the proposed nominal LQRs with robust pole clustering still preserve optimality with respect to a specific quadratic cost function. Comparison of our method with some other existing methods is carried out using illustrative examples.

(10). Wang, S.G. and L.S. Shieh, "Robust Optimal Pole-Clustering in a Vertical Strip and Disturbance Rejection for Uncertain Lagrange's Systems," *Dynamics and Control*, Vol. 5, pp. 295-312, 1995.

Abstract

This paper presents an approach to design a state-feedback robust control law for uncertain Lagrange's systems such that the designed closed-loop systems have the properties of robust pole-clustering within a vertical strip and disturbance rejection with an H_∞-norm constraint. This approach is based on solving an algebraic Riccati equation with the adjustable scalars and prespecified parameters. The uncertainties considered include both unstructured and structured uncertainties in the system and the input matrices. Also, a constraint is established to verify that the proposed robust LQRs preserve H_2 optimality with respect to a specific quadratic cost function.

(11). Hsieh, C.S., C. Hwang and L.S. Shieh, "A Direct Trunction Method for Biased Reduced-Order Models," *Proc. Natl. Sci. Counc. ROC(A)* , Vol. 19, No. 4, pp. 329-333, 1995.

Abstract

A direct truncation method is presented for obtaining stable biased reduced-order models such that combinations of retained time moments and Markov parameters may be varied, and the stability can be preserved. The method is very simple and has greater flexibility than most model reduction methods. A family of reduced-order models can be easily generated by varying a free parameter in the denominator of a reduced-order model. The stability of the reduced-order models, which have order not greater than nine, can be easily retained.

(12). Wang, S.G., L.S. Shieh and J. W. Sunkel, "Robust optimal pole-placement in a vertical strip and disturbance rejection," *Int. J. Systems Sci.* , Vol. 26, No. 10, pp. 1839-1853, 1995.

Abstract

This paper presents a linear quadratic regulator (LQR) for robust closed-loop pole-placement within a vertical strip, and disturbance rejection with an H_∞-norm constraint for the uncertain linear systems. The concerned systems cover both matched and mismatched uncertain linear systems with unstructured or structured uncertainties existing in both the system and input matrices. A set of tuning parameters is incorporated for some flexibility in finding a solution to the algebraic Riccati equation, and a controller dain parameter is selected for robust pole clustering. Also, a constraint is established to verify whether the proposed robust LQRs preserve H_2 optimality with respect to a specific quadratic cost function.

(13). Shieh, L.S., X. Zou and N.P. Coleman, "Digital interval model conversion and simulation of continuous-time uncertain systems," *IEE Proceedings D. Control Theory and Applications*, , Vol. 142, No. 4, pp. 315-322, 1995.

Abstract

The paper deals with the problem of converting a continuous-time uncertain linear system to an equivalent discrete-time uncertain model and its digital simulation. The system matrices characterising the state-space representation of the original uncertain system are assumed to be interval matrices. The geometric series method together with interval arithmetic is employed to obtain the approximate discrete-time interval models. A new technique is developed to estimate the modelling errors. These modelling errors are used to modify the approximate interval models obtained via the interval geometric-series method. The resulting interval models (the enclosing interval models) are able to tightly enclose the exact uncertain model. Also their approximate discrete-time interval solutions are able to tightly enclose the exact interval solution of the continuous-time uncertain state-space equation. The proposed digital uncertain models can be used for digital simulation and digital design of continuous-time uncertain systems.

(14). Shieh, L.S., J. Gu and J.S.H. Tsai, "Model conversions of uncertain linear systems using a scaling and squaring geometric series method," *Circuits Systems Signal Process* , Vol. 14, No. 4, pp. 445-463, 1995.

Abstract

This paper proposes a scaling and squaring geometric series method along with the inverse-geometric series method for finding discrete-time (continuous-time) structured uncertain linear models from continuous-time (discrete-time) structured uncertain linear systems. The above methods allow the use of well-developed theorems and algorithms in the discrete-time (conttinuous-time) domain to indirectly solve the continuous-time (discrete-time) domain problems. Moreover, these methods enhance the flexibility in modeling and control of a hybrid composite

system. It has been shown that the commonly used bilinear approximation model is a specific class of the proposed geometric series model.

(15). Tsai, J.S.H., C.P. Fan and L.S. Shieh, "Implementation of State-feedback Control Law for Singular Systems Via an Input-Output Feedback Structure," *International Journal of Systems Science* , Vol. 26, No. 11, pp. 2139-2158, November, 1995.

Abstract

The paper presents a state-feedback design methodology for singular systems and proposes an input-output feedback structure for implementation of the designed state-feedback control law. First, the state-space representation of a singular system is decomposed into a reduced-order regular subsystem and a fast subsystem which include impulsive modes and infinite nondynamic modes, in which the states of impulsive modes cannot be estimated by means of the conventional state observer. Next, a state-feedback design concept is introduced to eliminate all impulsive modes of the fast subsystem and to optimally place the eigenvalues of the regular subsystem within a specific region. Finally, the obtained state-feedback control law is implemented by the proposed stable input-output feedback structure.

(16). Shieh, L.S., J. Gu and J.S.H. Tsai, "Robust Digital Redesign of Uncertain Linear Systems Using the Interval Bilinear Approximation Method," *Control Theory and Advanced Technology* , Vol. 10, No. 4, Part 4, pp. 1667-1688, November, 1995.

Abstract

This paper presents an interval bilinear approximation method to convert a continuous-time state-feedback robust control law to an equivalent discrete-time counterpart for robust digital control of a continuous-time uncertain system. For performing the digital redesign of a continuous-time uncertain system, the given continuous-time uncertain system is converted to a tightly bounded equivalent discrete-time uncertain model using the bilinear approximation method aided by the interval arithmetic operations. Also, utilizing the interval bilinear approximation method, the continuous-time robust control law is converted to an equivalent discrete-time interval control law. Applying this digitally redesigned control law, the states of the digitally controlled sampled-data uncertain system closely match those of the original continuous-time controlled uncertain system. A numerical example is presented to demonstrate the effectiveness of the proposed method.

(17). Shieh, L.S., I.C. Lin and J.S.H. Tsai, "Design of PWM Controller for Sampled-Data System Using Digitally Redesigned PAM controller," *Proceedings of IEE, Part D, Control Theory and Applications* , Vol. 142, No. 6, pp. 654-660, November, 1995.

Abstract

Two issues are addressed: Digital redesign of a continuous-time system with an input time delay, and translation of the pulse-amplitude modulated (PAM) controller obtained by a newly proposed digital redesign method into an equivalent pulse-width modulated (PWM) controller. A tuning parameter is introduced into the PAM controller so that the digitally controlled sampled-data states closely match the original continuous-time input time-delay states. Also, the principle of equivalent areas is applied to convert the newly developed PAM controller into an equivalent PWM controller so that the PWM controlled states closely match the PAM controlled states for a relatively longer sampling period. Two illustrative examples are provided to demonstrate the effectiveness of the proposed method.

(18). Shieh, L.S., W.M. Wang, and J.S.H. Tsai, "Digital Modelling and Digital Redesign of Sampled-Data Uncertain Systems," *Proceedings of IEE, Part D., Control Theory and Applications*, Vol. 142, No. 6, pp. 585-594, November, 1995.

Abstract

A new method for digital model conversion of a continuous-time uncertain system and a new digital redesign method for robust control of a sampled-data uncertain system are presented. The concept of the principle of equivalent areas together with interval arithmetic is utilised for finding the discrete-time uncertain model and the digital robust control law from the continuous-time uncertain state equation and the analogue robust control law, respectively. Using the newly digitally redesigned controllers, the resulting dynamic states of the digitally controlled sampled-data uncertain systems are able to closely match those of the original analogously controlled continuous-time uncertain systems for a relatively longer sampling period.

(19). Shieh, L.S., X. Zou and N.P. Coleman, "Digital Interval Modelling and Hybrid Control of Uncertain Systems," *IEEE Trans. Industrial Electronics* , Vol. 43, No. 1, pp. 173-183, February, 1996.

Abstract

This paper addresses two issues: 1) converting a continuous-time uncertain system to an equivalent discrete-time interval model; and 2) constructing a robust digital control law from a robust analogue control law for hybrid control of sampled-data uncertain systems. The system matrices characterizing the state-space representation of the original continuous-time uncertain systems are assumed to be interval matrices. The Pade approximation method together with a geometric-series approximation method is employed to obtain the generalized enclosing discrete-time interval models. The generalized enclosing interval models are able to tightly enclose the exact discrete-time

uncertain model, and can be utilized for digital simulation and digital design of continuous-time uncertain systems.

A new family of digitally redesigned interval controllers is constructed from a continuous-time robust controller for robust digital control of continuous-time uncertain systems. Using the newly digitally redesigned interval controllers, the dynamic states of the digitally controlled sampled-data uncertain systems are able to closely match those of the original analogously controlled continuous-time uncertain systems for a relatively longer sampling period.

(20). Shieh, L.S., J. Gu and J.S.H. Tsai, "Model Conversions of Uncertain Linear Systems Via the Interval Pade Approximation Method," *Circuits, Systems, and Signal Processing* , Vol. 15, No. 1, pp. 1-22, 1996.

Abstract

This paper presents a new interval Pade approximation method to convert a continuous-time (discrete-time) uncertain linear system to an equivalent discrete-time (continuous-time) uncertain model via interval arithmetic operations. Based on the inclusion theorem related to the interval arithmetic, the interval Pade's approximants and their associated interval error matrices with interval arguments are obtained via the Pade's approximants and their associated error matrices with degenerate (real) arguments, respectively. Tighter error bounds of various approximate uncertain models with respect to their exact uncertain models are determined and used to modify the obtained Pade's approximants, so that the resulting approximate uncertain models are able to tightly enclose the original uncertain systems. Thus, the analysis and design of the original uncertain systems can be indirectly carried out using the converted uncertain models in either the continuous-time or the discrete-time domain.

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